

School Name: _____

Team Name: _____

2015 SECME Regional
Mathematics Contest
Middle School
Exam, Form: A

Date: February 14, 2015

Answer the questions in the space provided below. Show your work on the scrap paper you have been provided. Make sure you clearly label the work for each problem. Include units when necessary.

1. If the perimeter of a regular polygon is 35 feet and has side lengths that are whole numbers bigger than 1, how many sides does the polygon have and what is the length of the sides? (There are two possible answers to this question. You only need to provide one to receive full credit.)

2. If you can buy 8 pounds of peanuts for \$10, how many pounds of peanuts can you buy for \$15?

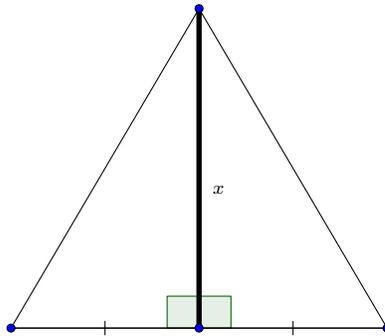
3. What is the equation of the line passing through the point $(0, 2)$ and perpendicular to $y = \frac{1}{2}x + 2$?
(Write your answer in slope-intercept form.)

4. Suppose Robert competes in a math competition that gives the winner \$500 plus \$15 for every question the winner answers correctly. Assume that Robert wins the competition and is given \$665. If the competition had 15 questions, how many questions did he answer incorrectly?

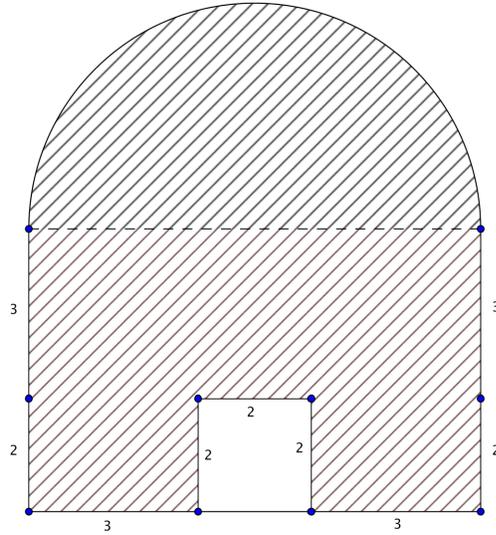
5. Suppose you spend \$49 on a combination of gummy bears and lollipops. Gummy bears are \$4 per pound and lollipops are \$3 per pound. If the number of pounds of gummy bears you buy is 4 more than 2 times the number of pounds of lollipops, then how many pounds of each do you buy?

6. Suppose you buy a yellow submarine on sale for \$10,000. If you received an 18% discount, was was the original cost of the yellow submarine? Round your answer to the nearest cent.

7. In the diagram below, we have an equilateral triangle with side lengths 10 inches. Find x (the length of the bold line). (Give an exact answer in simplified form.)



8. Find the area of the shaded region in the diagram below. The units are in feet.



9. You are practicing archery and you measure how close to the bull's eye the arrow is. For the first round, the arrow is 8 cm away. For the second round, the arrow is half the distance from the first round. For the last round, the arrow is one cm further from the bull's eye than round 2. What is the average distance between the arrow and the bull's eye? (Write your answer using fractions.)

10. Simplify $\left(\frac{1}{8} + \frac{5}{12}\right)\left(\frac{5}{6}\right)$.

11. Find the missing number in the sequence 2, -5, 8, ____, 14, -17, 20, ...

12. Given that 2.54 cm is 1 inch, convert 5 meters into inches (round your answer to two decimal places).

13. There are two values of x for which $x^2 + 4x + 3 = (x + 3)(x + 1)$ will equal zero. What is the larger of the two x values?

14. Given the following set of numbers, what number do you get when you subtract the mode from the range?

10, 15, 2, 12, 17, 48, 12, 36

15. The radius of the globe in front of Universal Studios is six more than three times the number of letters in the word “Orlando” (measured in feet). Given that the volume of a sphere is $\frac{4}{3}\pi r^3$, find the exact volume of the Universal Studios globe.

(Note: This question is not based on real data.)

16. Consider the line $y = 3x + 5$.

- (a) What is the slope of any line parallel to this line?
- (b) What is the slope of any line perpendicular to this line?

17. Simplify $(4xy + y - 3x + 12) - (xy + 2x - 4y + 4)$.

Answer Key for Exam A

Answer the questions in the space provided below. Show your work on the scrap paper you have been provided. Make sure you clearly label the work for each problem. Include units when necessary.

1. If the perimeter of a regular polygon is 35 feet and has side lengths that are whole numbers bigger than 1, how many sides does the polygon have and what is the length of the sides? (There are two possible answers to this question. You only need to provide one to receive full credit.)

There are two possible answers for this.

- 5 sides each with length 7 feet
- 7 sides each with length 5 feet

2. If you can buy 8 pounds of peanuts for \$10, how many pounds of peanuts can you buy for \$15?

$$\frac{8 \text{ lbs}}{\$10} = \frac{x \text{ lbs}}{\$15}$$

This gives $8(15) = 10x$ which means $x = 12$ pounds.

3. What is the equation of the line passing through the point $(0, 2)$ and perpendicular to $y = \frac{1}{2}x + 2$? (Write your answer in slope-intercept form.)

The slope of the line is $m = -2$. Using the point slope formula, we have

$$y - y_0 = m(x - x_0)$$

$$y - 2 = -2(x - 0)$$

$$y = -2x + 2$$

So the equation of the line is $y = -2x + 2$.

4. Suppose Robert competes in a math competition that gives the winner \$500 plus \$15 for every question the winner answers correctly. Assume that Robert wins the competition and is given \$665. If the competition had 15 questions, how many questions did he answer incorrectly?

$$15 - \frac{\$665 - \$500}{\$15} = 15 - \frac{\$165}{\$15} = 15 - 11 = 4.$$

He answered 4 questions incorrectly.

5. Suppose you spend \$49 on a combination of gummy bears and lollipops. Gummy bears are \$4 per pound and lollipops are \$3 per pound. If the number of pounds of gummy bears you buy is 4 more than 2 times the number of pounds of lollipops, then how many pounds of each do you buy? Let G be the number of pounds of gummy bears purchase and L be the number of pounds of lollipops you purchase. Then we have the following system of linear equations:

$$4G + 3L = 49$$

$$4 + 2L = G.$$

Then

$$4G + 3L = 49$$

$$4(4 + 2L) + 3L = 49$$

$$16 + 8L + 3L = 49$$

$$11L = 33$$

$$L = 3$$

Then, $4 + 2L = 4 + 2(3) = 4 + 6 = 10 = G$.

Thus, you purchase **3 pounds of lollipops and 10 pounds of gummy bears.**

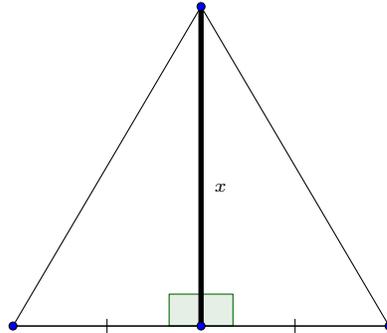
6. Suppose you buy a yellow submarine on sale for \$10,000. If you received an 18% discount, was was the original cost of the yellow submarine? Round your answer to the nearest cent. **Let x be the original cost of the yellow submarine. Then**

$$10000 = .82x$$

$$\$12,195.12 \approx x.$$

The yellow submarine originally cost \$12,195.12.

7. In the diagram below, we have an equilateral triangle with side lengths 10 inches. Find x (the length of the bold line). (Give an exact answer in simplified form.)



The bold line creates two right triangles. Each have legs length x and $\frac{10}{2} = 5$ and hypotenuse 10. Using Pythagorean theorem, we have

$$x^2 + 5^2 = 10^2$$

$$x^2 + 25 = 100$$

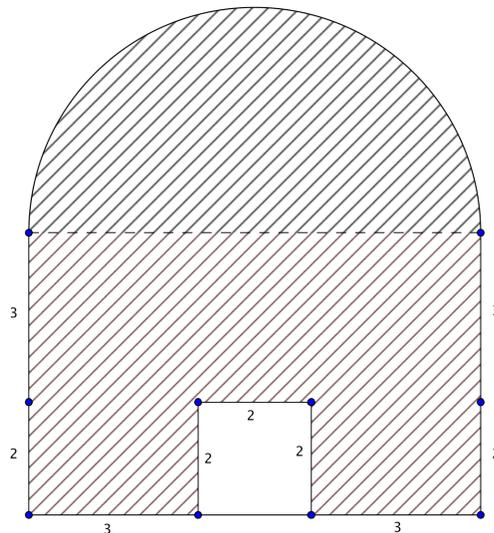
$$x^2 = 75$$

$$x = \sqrt{75}$$

$$x = 5\sqrt{3}.$$

Therefore, x is $5\sqrt{3}$ inches.

8. Find the area of the shaded region in the diagram below. The units are in feet.



The area of the semicircle is $\frac{1}{2}\pi(4)^2 = 8\pi$. The area of the rectangular region is $40 - 4 = 36$. Then the area of the shaded region is $\underline{36 + 8\pi \text{ ft}^2}$.

9. You are practicing archery and you measure how close to the bull's eye the arrow is. For the first round, the arrow is 8 cm away. For the second round, the arrow is half the distance from the first round. For the last round, the arrow is one cm further from the bull's eye than round 2. What is the average distance between the arrow and the bull's eye? (Write your answer using fractions.) **The average is $\frac{8 + 4 + 5}{3} = \frac{17}{3} \text{ cm} = 5\frac{2}{3} \text{ cm}$ (either proper or improper fraction is okay).**
10. Simplify $\left(\frac{1}{8} + \frac{5}{12}\right)\left(\frac{5}{6}\right)$.

$$\begin{aligned}\left(\frac{1}{8} + \frac{5}{12}\right)\left(\frac{5}{6}\right) &= \left(\frac{3 + 10}{24}\right)\left(\frac{5}{6}\right) \\ &= \frac{13}{24}\left(\frac{5}{6}\right) \\ &= \frac{65}{144}\end{aligned}$$

11. Find the missing number in the sequence 2, -5, 8, ____, 14, -17, 20, ... **The missing number is -11.**
12. Given that 2.54 cm is 1 inch, convert 5 meters into inches (round your answer to two decimal places). $\frac{5 * 100 * 1}{1 * 2.54} \approx 196.85 \text{ in}$
13. There are two values of x for which $x^2 + 4x + 3 = (x + 3)(x + 1)$ will equal zero. What is the larger of the two x values? **The two x values are $x = -1, -3$. The larger of the two is $x = -1$.**
14. Given the following set of numbers, what number do you get when you subtract the mode from the range?

10, 15, 2, 12, 17, 48, 12, 36

The range is $48 - 2 = 46$ and the mode is 12. Therefore, when you subtract the mode from the range, you get $46 - 12 = 34$.

15. The radius of the globe in front of Universal Studios is six more than three times the number of letters in the word "Orlando" (measured in feet). Given that the volume of a sphere is $\frac{4}{3}\pi r^3$, find the exact volume of the Universal Studios globe.
(Note: This question is not based on real data.) **There are seven letters in "Orlando." Then $3 * 7 + 6 = 27$. Then the volume is $\frac{4}{3}\pi(27)^3 = 26244 \text{ ft}^3$.**
16. Consider the line $y = 3x + 5$.
- (a) What is the slope of any line parallel to this line?
(b) What is the slope of any line perpendicular to this line?
- (a) **The slope is 3.**
(b) **The slope is $-\frac{1}{3}$.**
17. Simplify $(4xy + y - 3x + 12) - (xy + 2x - 4y + 4)$. The answer is **$3xy + 5y - 5x + 8$** .